The Estimation of the Operational Capacity of the Logistic System According to the Level of the Functionality Status of its Constituent Organizations

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Abstract

The operational capacity represents one of the most important characteristics of the logistic system, from the perspective of its reliability. The present paper describes, concisely, in its first part, the theoretic foundations of the analysis and of the evaluation of a logistic system from the point of view of the reliability function and in its second part there are introduced two models of a system which consists of n *components, which is operational in the circumstances in which at least* k *components, out of the* n *components, are operational. In the two models it's analyzed the functionality of the logistic system, at its two levels –the upstream level and the downstream level-, from the point of view of the level of the operational status of the suppliers, respectively of the distribution centers. In this context, it results that a special influence is exerted by the number of the logistics centers, as key elements of the logistic system. Finally, it's carried out an evaluation of the way in which the operational capacity of the introduced logistic system is determined by the functionality statuses of the organizations from which the system is composed.*

Keywords: *operational capacity, logistic system, operational status, system reliability, function-structure.*

JEL classification: C02, L81, L14, L22, M11.

1. Introduction

A system represents an array of components, subsystems and assemblies and subassemblies, arranged in a certain configuration, in order to fulfill a certain function, at a certain level of performance and reliability. The type of the components, their number and the quality and the way in which they interconnect

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in a system are characteristics with directly influence the system's reliability. In the early '60s were promoted a series of spatial concepts and spatial planning, which have resulted in the reference documents over time (Nichersu & Iacoboaea, 2011).

From the technical point of view, the reliability represents the probability for a system or a component to fulfill, in a proper way, in a certain period of time, the function assigned to the system or the component, in the operational conditions for which it was created.

The reliability of the logistic system, as a whole constituted out of several organizational entities that collaborate with one another, represents the probability for it to detain the operational capacity that is necessary to fulfill the orders received from its clients, in the circumstances of an environment in which certain factors of endogenous and exogenous nature operate.

The reliability of the logistic system is tightly related to the individual or cumulated level of the operational status of the component organizational entities. An organization which plays an important role inside the logistic system and has in the same time a low level of the operational status will determine a significant decline of the **reliability of the system, respectively of the operational capacity of it.** The same influence will be exerted to the logistic system in the case in which several organizational entities which compose it, but which have a smaller signification, record simultaneously a low level of the operational status.

1. Methodological issues regarding the estimation of the operational capacity of a logistic system

The whole operational capacity of a logistic system means performance, through the continuous improving of the quality of the services offered to the clients and the products offered to those clients, according to their requirements. From this perspective**, the operational capacity, respectively the reliability** represents a "anchor of quality" (King & Jewett, 2010), having in mind that, any lowering of the performances is the equivalent, in the consumer's perception, of a lowering in quality.

The low level of the quality of the services offered, respectively of the products supplied to the clients, determined by the diminishing of the operational capacity of the logistic system, leads in the majority of cases to increases in the operating costs and the associated costs, through:

- the significant decline of the brand's reputation and of consumer's trust;
- the significant decreasing of the number of clients and even the loss of the whole business;
- the necessity of the immediate development of an increased volume of maintenance operations of the system;
- the payment of some penalties or harms;
- the necessity of elimination of some eventual material losses.

A binary logistic system is a logistic system in which both itself, and the organizations from which it is constituted (its components) can be either in an operational status, or in an unoperational status. If x_i represents the operational status of a component *i*, for $1 \le i \le n$ and

$$x_i = \begin{cases} 1 \text{ if component i is operational} \\ 0 \text{ if component i isnt operational} \end{cases}$$

then the vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ represents the status vector of all the components of the logistic system. In these conditions, the status of the logistic system is a binary hazard variable, completely determined by the statues of its components.

If $\square_s$ is the state of the logistic system, then:

$$\phi_s = \begin{cases} 1 \text{ if the logistic system is operational} \\ 0 \text{ if the logistic system isnt operational} \end{cases}$$

If the operational statuses of all the component organizations are known, then the operational status of the logistic system is also known. Therefore, the operational status of the logistic system is a function that is determined by the operational statuses of the organizations out of which the system is constituted, an aspect which is expressed through the ratio:

$$\phi_s = \phi(\mathbf{x}) = \phi(x_1, x_2, \ldots, x_n)$$
$$i = 1, 2, \ldots, n$$

in which $\phi(\mathbf{x})$ represents the *structure-function of the logistic system.*

Usually, when we analyze the structure of a logistic system, we separate initially (for an ulterior analysis) or we even ignore the components which don't play a direct role in its functioning. The remaining components, in order to be profoundly analyzed, are called *relevant*, the others, obviously, being called *irrelevant.*

A component *i* is relevant if:

$\phi(1_i, \mathbf{x}) = \phi(0_i, \mathbf{x})$, for all $(*_{\mathrm{i}}, \mathbf{x})$,

in which:

$(1_{\mathrm{i}}, \mathbf{x})$ - the status vector in which the component *i* equals 1;

$(0_{\mathrm{i}}, \mathbf{x})$ -the status vector in which component *i* equals 0;

$(*_{\mathrm{i}}, \mathbf{x})$ - the status vector in which component *i* equals 0 or 1.

A binary system which is composed out of *n* components is *coherent* if it has a structure-function to meet the following conditions (Pham, 2003):

1. $\phi(\mathbf{x})$ is a function which isn't increasing in each argument x_i, $i = 1, 2, \ldots, n$;

2. there is a vector **x**, in such a way that $0 = \phi(0_i, \mathbf{x}) < \phi(1_i, \mathbf{x}) = 1$;

3. $\phi(0) = 0$ şi $\phi(1) = 1$.

The condition number 1 proves to us that $\phi(x)$ is a monotonically increasing function in each argument, the condition number 2 representing the condition that each component from the system to be relevant for its performance, and the condition number 3 shows us that the system is operational when all its components are operational and the same way around.

It's called logistic system *k from n:G*, which is noted down as *k/n:G*, that system which is formed from n organizations, which is operational, if and only if at least k organizations which are components of *n* are operational.

Analogously, it's called a logistic system *k from n:F*, which is noted down *k/n:F*, that system which is formed out of *n* organizations, which isn't operational if and only if at least *k* organizations from *n* aren't operational.

A logistic system formed from *n* components, which are serially structured, can be considered to be a *serial system* if, in order to be operational, the whole n components are operational. In the same time, if a component becomes unoperational, then the whole system will become unoperational.

If x_i is the event through which component i is operational and R_i represents the reliability of the component i, then the reliability R_{Ss} of the system *S* with the serial structure and independent components will be given by the ratio (Dhillon, 2005; Epstein & Weissman 2008; Langford 2007):

$$R_{Ss} = P(x_1) \cdot P(x_2) \cdot \ldots \cdot P(x_n) = \prod_{i=1}^{n} P(x_i) \tag{1}$$

where $P(x_i)$ represents the probability of the appearance of the event x_i, $i = 1, 2, \ldots, n$.

If $R_i = P(x_i)$, the ratio (1) becomes:

$$R_{Ss} = \prod_{i=1}^{n} R_i \tag{2}$$

In these conditions, the structure-function of the series system will be given by the ratio:

$$\phi(\mathbf{x}) = \prod_{i=1}^{n} x_i = \min_{1 \le i \le n} \{x_1, x_2, \ldots, x_n\} \tag{3}$$

Therefore, a logistic system with a serial structure is operational if and only if all the organizations from which it is composed are operational, respectively if the status of a logistic system is $\phi_s = 1$, if and only if $x_i = 1$, for $i = 1, 2, \ldots, n$. Therefore, a system formed of *n* components that act in the same time, has a structure with parallel components, in short is a parallel system, if an unoperational component of it doesn't determine the lack of functionality of the whole system. The system becomes unfunctional if its components are unfunctional.

If $\overline{x}_i$ is the event through which component i is unoperational and R_{Sp} represents the reliability of the system S with a structure in parallel and independent component, the probability, Q_S, that this kind of system to be unoperational will be Dhillon, 2005; Epstein & Weissman 2008; Langford 2007):

$$Q_S = P(\overline{x}_1)\cdot P(\overline{x}_2)\cdot\ldots\cdot P(\overline{x}_n) = \prod_{i=1}^{n} P(\overline{x}_i)$$

(4)

Where $P(\overline{x}_i)$ represents the probability of appearing $\overline{x}_i$, $i = 1, 2, \ldots, n$. Therefore it results that:

$$R_{Sp} = 1 - Q_s = 1 - \prod_{i=1}^{n} P(\overline{x}_i) = 1 - \prod_{i=1}^{n} \left[1 - P(x_i)\right] = 1 - \prod_{i=1}^{n} (1 - R_i) \quad (5)$$

The structure-function of the parallel system will be calculated by the ratio:

$$\phi(\mathbf{x}) = 1 - \prod_{i=1}^{n} (1 - x_i) = \coprod_{i=1}^{n} x_i = \max_{1 \le i \le n} \{x_1, x_2, \ldots, x_n\} \quad (6)$$

Therefore, a logistic system with a parallel structure is unoperational if and only if all the organizations from its structure are unoperational, respectively the status of the logistic system is $\phi_s = 0$, if and only if $x_i = 0$, for $i = 1, 2, \ldots, n$. In the real circumstances, both the logistic system and the organizations from which the logistic system is composed can be in more then two operational statuses, according to the operational capacity it has at a certain moment.

For example, the logistic system or a component organization can be, in certain moments, in a complete operational state, when the operational capacity is maximal, in partial operational status, with a operational capacity of 80%, in a partial operational status, with an operational capacity of 50% or in an unoperational status. This way, through this example we defined four levels of the operational statuses. Therefore, a binary logistic system becomes a logistic system with several statuses, if this and the component organizations extend the number of statuses from $\{0,1\}$ at $\{0,1,\ldots,M\}, M > 1, M \in \mathbb{N}$, the level M expressing the completely operational status.

In a logistic system with several statuses, the status is which the system enters is determined by the status in which each component organization enters.

The performance of the entire logistic system can be evaluated through the distribution of its status, given according to the distribution function of the probabilities or the reliability-function, which show us its operational capacity.

Analogously, the binary system, a system with several statuses and n components, can be defined as a *coherent* system, if it has a structure-function to comply with the following conditions (Pham, 2003):

1. $\phi(\mathbf{x})$ is a monotonically increasing function in each argument;

2. There is a vector **x**, so that $0=\phi\left(0_i,\mathbf{x}\right)<\phi\left(M_i,\mathbf{x}\right)=1$, for each $i=1,2,\ldots,n$; (7)

3. $\phi\left(\mathbf{j}\right)=j$ for $j=1,2,\ldots,M$.

Adapting the ratios (3) and (6) for a logistic system with several statuses *k/n:G*, with a probability to be in the status j or to be superior to it, $R\left(k,n;j\right)$, it will be:

1. $\phi\left(\mathbf{x}\right)=\min\limits_{1\le i\le n} x_i$ (8)

Therefore, the operational capacity of a logistic system with several serial statuses is determined by the organization with the lowest level of the operational status in its structure.

2. $\phi\left(\mathbf{x}\right)=\max\limits_{1\le i\le n} x_i$ (9)

Therefore, the operational capacity of a logistic system with several parallel statuses is determined by the organization with the highest level of the operational status from its structure.

The evaluation of the operational capacity of a logistic system with several statuses can be made with the following ratios:

$$R_{s,j}=\prod_{i=1}^{n}P_{i,j}\text{ , }1\le j\le M\text{ , for the series system} \tag{10}$$

$$Q_{s,j}=\prod_{i=1}^{n}Q_{i,j}\text{ , }1\le j\le M\text{ , for the parallel system} \tag{11}$$

In real circumstances, the logistic systems with several statuses differ accordingly to the level of the operational status of the system. For example, if we have a logistic system which has in its structure 3 organizations in which exist 4 possible operational statuses, we can identify the following cases:

- for the system to be placed at the third level of the operational status, it has to have a 3/3:G structure and for that at least three organizations must be in the third operational status, for the logistic system to be in this operational status;
- for the system to be placed at least at the second level of the operational status, it has to have a 2/3:G structure and for that at least two organizations must be in the second operational status or in a superior operational status, for the logistic system to be in the second operational status or in the superior operational status;
- for the system to be placed at least at the first level of the operational status, it has to have a 1/3:G structure and for that at least an organization from which it's composed must be in the second operational status or in a superior operational status, for the whole logistic system to be in the first operational status or in the superior operational status;
- at the zero level the system isn't an operational one, all the component organizations being unoperational.

A logistic system which is composed of n organizations is called a *generalized system with several statuses*, denoted *k/n:G*, if it has the structure-function,

$$\phi(\mathbf{x}) \geq j, (j = 1, 2, \ldots, M) \tag{12}$$

in the circumstances in which there is an integer value *l*, $(j \leq l \leq M)$, in such a way that k_l components of the system to be in the operational status *l*, or in superior statuses. In such a system, the value k_j is different from a status j to another, $(j \leq l \leq M)$, which means that the structure of a system with several statuses differs according to the level of the system's value (Pham, 2003).

Two important cases are distinguished:

I. In the case in which $k_1 \leq k_2 \leq \ldots \leq k_M$, we have a *k/n:G logistic system with several statuses, ascending.*

For the logistic system to be in a superior operational status, j, it's necessary that a large number of organizations from which it's composed to be in this operational status j or in operational statuses which are superior to it. In this case, it must be applied a strategy of increasing the number of organizations which are in this target-operational status or in statuses which are superior to it.

For a *k/n:G*, ascending, logistic system with several statuses, the ratio (12) is transformed into:

$\phi(\mathbf{x}) \geq j$, if and only if at least k_j components are in *j* operational status or in statuses superior to $j\left(x_i \geq j\right)$ (13)

When all the components are independent and have the same probability distribution, the probability that logistic system to be in j operational status, or in a superior status to it, $R_{s,j}$ is defined by the ration of the binomial distribution (Epstein & Weissman, 2008):

$$R_{s,j} = \sum_{k=k_j}^{n} C_n^k \cdot P_j^k \cdot Q_j^{n-k} \tag{14}$$

where:

$R_{s,j} = \Pr\left(\phi(\mathbf{x}) \geq j\right)$

C_n^k - the ratio of the mathematical components;

P_j - the probability of a component from the system to be in j status or in a status superior to it;

$Q_j = \left(1 - P_j\right)$ - the probability that a component of the system not to be in j status or in a status superior to it;

The probability that the logistic system to be in the j status, is defined by the relationship:

$$\mathrm{r}_{\mathrm{s,j}} = \mathrm{R}_{\mathrm{s,j}} - \mathrm{R}_{\mathrm{s,j+1}} \tag{15}$$

and $R_{s,0} = 1$ şi $r_{s,j} = \Pr\left(\phi(\mathbf{x}) = j\right)$.

II. In the case in which $k_1 \geq k_2 \geq \ldots \geq k_M$, we have a *descending. k/n:G logistic system with several statuses.*

In this case, the logistic system is in a superior operational status, j, in the cases in which a reduced number of organizations from which it's composed are in this operational status or in operational statuses superior to it. For the logistic system to be in the superior status we want, it must be carried out a strategy of restricting the collaboration with a large number of organizations which have low operational capacities and intensifying the collaboration with a reduced number of organizations which have high operational capacities. The logistic system is at M level, if at least k_M component organizations are at that level. In the same time, the logistic level is at least at *M-1*, if at least k_{M-1} component organizations are at this level or at a superior level or at least k_M component organizations are at M level.

By generalization, it can be said that this kind of logistic system is in a j operational status or in superior statuses to it $(1 \leq j \leq M)$, if:

- at least k_j components are in *j* status or in superior statuses superior to it;
- at least k_{j+1} components are in j+1 status or in statuses superior to it;
- at least k_{j+2} components are in j+2 status or in superior statuses to it;
- at least k_M components are in M status.

For a *k/n:G* descending, logistic system to be in several statuses, the ratio (12) is transformed into:

$\phi(\mathbf{x}) = j$, if and only if at least k_j component organizations are in j operational status or in superior operational statuses to it and the majority of the component organizations k_{l-1} are in the operational status l or in operational statuses which are superior to it, for

$$l = j+1, j+2, \ldots, M \text{ , unde } j = 1, 2, \ldots, M \tag{16}$$

When all the components are independent and they have the same status probability distribution, p_j, the probability that a logistic system to be in j operational status is defined by the ratio (Pham, 2003):

$$r_{s,j} = \sum_{k=k_j}^{n} C_n^k \cdot \left(\sum_{m=0}^{j-1} p_m \right)^{n-k} \cdot \left(p_j^k + \sum_{l=j+1, k_l>1}^{M} \beta_l(k) \right) = \sum_{k=k_j}^{n} C_n^k \cdot Q_j^{n-k} \cdot \left(p_j^k + \sum_{l=j+1, k_l>1}^{M} \beta_l(k) \right) \tag{17}$$

where:

p_j^k represents the probability that all *k* components to be exactly in j status;

$\beta_l(k)$ is the probability that:

- at least 1 and at most $k_l - 1$ components to be in l status;

- at most $k_u - 1$ components to be in status u, for $j+1 \le u < l$;
- a number of k components to be in *j* and *l* status.
- the whole system to be in *j* status.

This probability can be calculated with the relationship (Pham, 2003):

$$\beta_l(k) = \sum_{i_1=1}^{k_l-1} C_k^{i_1} \cdot p_l^{i_1} \cdot \sum_{i_2=0}^{k_{l-1}-1-I_1} C_{k-I_1}^{i_2} \cdot p_{l-1}^{i_2} \cdot \sum_{i_3=0}^{k_{l-2}-1-I_2} C_{k-I_2}^{i_3} \cdot p_{l-2}^{i_3} \cdot \ldots \cdot \sum_{i_{l-j}=0}^{k_{j+1}-1-I_{l-j-1}} C_{k-I_{l-j-1}}^{i_{l-j}} \cdot p_{j+1}^{i_{l-j}} \cdot p_j^{k-I_{l-j}} \quad (18)$$

in which:

$$I_{l-j-1} = \sum_{m=1}^{l-j-1} i_m \quad (19)$$

$$I_l = \sum_{m=1}^{l-j} i_m \quad (20)$$

The ratio (17) prove to us that, for a number of *k* components, *n-k* components can be at most in j operational status, the other k components being in j operational status or in operational statuses which are superior to it. This relationship sum up the probabilities that exactly k components to be in j operational status or in operational statuses which are superior to it, without them determine bringing the logistic system in a operational status superior to j status, for $k = k_j, k_{j+1}, \ldots, n$. Based on the theoretical elements, there are presented some numerical examples about the way of evaluation of the operational capacity of the logistic system according to the level of the operational status in which there are the organizational entities from which it's constituted.

The operational capacity of a logistic system, determined through the reliability-function, depends on the complexity of its structure, of the circumstances in which the component organizations develop their activities, of the risks and of the vulnerabilities at which them and the system as a whole are exposed, and the capabilities which these organizations have (human resources, means, capacities of maintenance, logistic infrastructure, etc.). In the same time, an important influence is exerted by the concurrent logistic systems which can interfere with the considered logistic system, through some component organizations which serve them in common, or due to other organizations that operate at the interface between systems, through the utilization of the same infrastructure or the same means (deposits, communication means, transport systems).

2. Practical aspects regarding the estimation of the operational capacity of a logistic system

We consider a logistic system with the structure from figure no. 1, formed of 14 organizational entities: 7 suppliers which cooperate with other logistic systems than the considered one, 2 logistic centers and 5 distribution centers.

In the first example, we consider that the supplier organizations and the logistic system which they belong to can be in the following situations:

✓ in the third operational stage, if the logistic system has the capacity of assuring 100% of the orders of deliveries to the clients, in the circumstances in which at least 5 suppliers deliver 100% of the production obtained;

✓ in the second operational stage, if the logistic system has the capacity of assuring 80% of the orders of deliveries to the clients, in the circumstances in which at least 6 suppliers deliver 80% of the production obtained;

✓ in the first operational stage, if the logistic system has the capacity of assuring 50% of the orders of deliveries to the clients, in the circumstances in which all the 7 suppliers deliver 50% of the production obtained;

✓ in zero operational stage, if the logistic system doesn't have the capacity of assuring at least 50% of the orders of delivery to the clients (due to the fact that the 7 suppliers can't deliver at least 50% of the production obtained or they can't achieve the production necessary to satisfy the requirement).

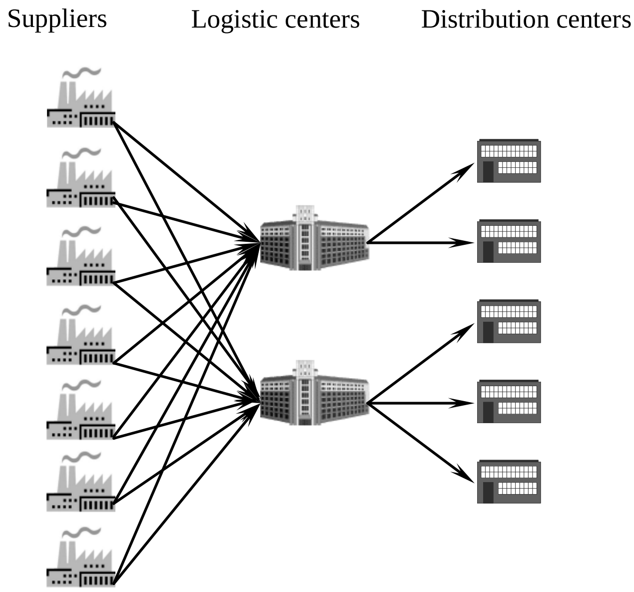

Figure 1 The organizational scheme of the logistic system

The situation of the suppliers according with their operational statuses are: =7; k_2=6; k_3=5, with $k_1 > k_2 > k_3$. We notice that in the case of a *k/n:G* system with several statuses, decreasing, respectively a *k/7:G* system, *decreasing.*

We consider the components of the logistic system independent, having the following probability of the status distribution

$$p_0 = 0,10; p_1 = 0,10; p_2 = 0,30; p_3 = 0,50.$$

The probabilities P_j that the organizations from the logistic system to be in the operational status j, or in operational statuses superior to it, are:

$$P_1 = \sum_{j=1}^{3} p_j = 0,90\,;\; P_2 = \sum_{j=2}^{3} p_j = 0,80\,;\; P_3 = p_3 = 0,50$$

The probabilities Q_j that the organizations from the logistic system to be at most in the operational status j are:

$$Q_3 = 1 - P_3 = 0,50;\; Q_2 = 1 - P_2 = 0,20;\; Q_1 = 1 - P_1 = 0,10\,.$$

We'll continue to determine the status probabilities of the logistic system.

In the third operational status: j=3, k_3=5.

Applying in the proper way the ratio (17) we obtain the probability that the system is in the third operational status:

$$r_{s,3} = \sum_{k=5}^{7} C_7^k \cdot Q_3^{7-k} \cdot p_3^k = \sum_{k=5}^{7} C_7^k \cdot 0,50^{7-k} \cdot 0,50^k = 0,2265$$

In the second operational status, we have: j=2, k_2=6, l=j+1=3.

Due to the fact that we have more than an organization in the superior operational status, ($k_l = k_{j+1} = k_3 = 5, k_3 > 1$), with the ratio (18) we'll have to calculate the probabilities $\beta_l(k) = \beta_{j+1}(k) = \beta_3(k)$, respectively $\beta_3(6)$ şi $\beta_3(7)$:

$$\beta_l(k) = \sum_{i_1=1}^{k_l-1} C_k^{i_1} \cdot p_l^{i_1} \cdot p_j^{k-I_{l-j}} = \sum_{i_1=1}^{k_3-1} C_k^{i_1} \cdot p_3^{i_1} \cdot p_2^{k-I_1}$$

$$\beta_3(6) = \sum_{i_1=1}^{4} C_6^{i_1} \cdot p_3^{i_1} \cdot p_2^{6-i_1} = \sum_{i_1=1}^{4} C_6^{i_1} \cdot 0,50^{i_1} \cdot 0,30^{6-i_1} = 0,1895$$

$$\beta_3(7) = \sum_{i_1=1}^{4} C_7^{i_1} \cdot p_3^{i_1} \cdot p_2^{7-i_1} = \sum_{i_1=1}^{4} C_7^{i_1} \cdot 0,50^{i_1} \cdot 0,30^{7-i_1} = 0,1098$$

So, the probability that a number of 6 suppliers to be in the operational statuses 2 and 3 is of 18,95%, and the probability that 7 suppliers to be in the same operational statuses is 10,98%, in the circumstances in which at least one supplier and at most 4 suppliers are in the operational status 3. We'll use the values obtained in order to determine the probability that the system to be in the second operational status, applying the ratio (17):

$$r_{s,2} = \sum_{k=k_2}^{7} C_7^k \cdot Q_2^{7-k} \cdot \left(p_2^k + \sum_{l=3}^{3} \beta_3(k) \right) = \sum_{k=6}^{7} C_7^k \cdot Q_2^{7-k} \cdot \left[p_2^k + \beta_3(6) + \beta_3(7) \right] =$$

$$= \sum_{k=6}^{7} C_7^k \cdot 0,20^{7-k} \cdot \left[0,30^k + \beta_3(6) + \beta_3(7) \right] = \sum_{k=6}^{7} C_7^k \cdot 0,20^{7-k} \cdot \left[0,30^k + 0,1895 + 0,1098 \right] = 0,3764$$

In the first operational status, we have: j=1, k_1=7, l=j+1=2. Due to the fact that we have more than an organization in the second operational status, $(k_l = k_{j+1} = k_2 = 6, k_2 > 1)$, with the ratio (18) we have to calculate the probability that $\beta_l(k) = \beta_{j+1}(k) = \beta_2(k)$. In the same time, in order to have more than an organization in the first operational status, $(k_{l+1} = k_{j+2} = k_3 = 5, k_3 > 1)$, we have to calculate with the same ratio (18) the probability $\beta_{l+1}(k) = \beta_{j+2}(k) = \beta_3(k)$. This way, we obtain:

$$\beta_2(k) = \sum_{i_1=1}^{k_2-1} C_k^{i_1} \cdot p_2^{i_1} \cdot p_1^{k-I_{l-j}} = \sum_{i_1=1}^{5} C_k^{i_1} \cdot 0,30^{i_1} \cdot 0,10^{k-I_1}$$

$$\beta_2(7) = \sum_{i_1=1}^{5} C_7^{i_1} \cdot 0,30^{i_1} \cdot 0,10^{7-i_1} = 0,0009$$

$$\beta_3(k) = \sum_{i_1=1}^{k_3-1} C_k^{i_1} \cdot p_3^{i_1} \cdot \left(\sum_{i_2=0}^{k_2-1-I_1} C_{k-I_1}^{i_2} \cdot p_2^{i_2} \cdot p_1^{k-I_2} \right)$$

$$\beta_3(7) = \sum_{i_1=1}^{4} C_7^{i_1} \cdot p_3^{i_1} \cdot \left(\sum_{i_2=0}^{4} C_{7-I_1}^{i_2} \cdot p_2^{i_2} \cdot p_1^{k-I_2} \right) = \sum_{i_1=1}^{4} C_7^{i_1} \cdot 0,50^{i_1} \cdot \left(\sum_{i_2=0}^{4} C_{7-i_1}^{i_2} \cdot 0,30^{i_2} \cdot 0,10^{7-i_1-i_2} \right) = 0,2996$$

This way, the probability that the 7 suppliers to be in the operational statuses 1 and 2 is of 0.09% (in the circumstances in which at least one supplier and at most 5 suppliers can be in the second operational status), while the probability that the 7 suppliers to be in the first and second operational statuses is of 29, 96% (in the circumstances in which at least one supplier and at most 5 suppliers can be in the second operational status), while the probability that the 7 suppliers to be in the first, second and third operational statuses is of 29, 96% (in the circumstances in which at least one supplier and at most 4 suppliers can be in the operational statuses 1 and 2). We'll use the obtained values in order to determine the probabilities that the system to be in the first operational status, applying the relationship (17):

$$r_{s,1} = \sum_{k=k_1}^{7} C_7^k \cdot Q_1^{7-k} \cdot \left(p_1^k + \sum_{l=2}^{3} \beta_l(k) \right) = \sum_{k=7}^{7} C_7^k \cdot Q_1^{7-k} \cdot \left[p_1^k + \beta_2(k) + \beta_3(k) \right] =$$

$$= \sum_{k=7}^{7} C_7^7 \cdot 0,10^0 \cdot \left[0,10^7 + \beta_2(7) + \beta_3(7) \right] = 0,10^7 + 0,0009 + 0,2996 = 0,3005$$

The obtained results are centralized in the table no. 1, in which they are presented also the probabilities that the logistic system to be in the operational status j or in the operational statuses superior to it.

The distribution of the probabilities of status of the logistic system from the perspective of the suppliers' operational status.

If in the preceding example we've analyzed the operational status of a logistic system from the point of view of the operational capacity of the organizations that supply the materials and the products, in the following lines we'll analyze, in another numerical example, the operational status of the same logistic system presented in the figure no. 1, but from the perspective of the operational capacity of the logistic centers and of the distribution centers. We consider that the logistic center and, implicitly, the logistic system can be in one of the following operational statuses:

- in the forth operational status, when at the downstream level of the logistic system the processing capacity of the delivery orders is maximal, all the logistic centers and all the distribution centers work at the maximal capacity;
- in the third operational status, when at the downstream level of the logistic system the processing capacity of the delivery orders is of 80%, respectively at least 2 logistic centers and 3 distribution centers work at 80% of the capacity;
- -in the second operational status, when at the downstream level of the logistic system the processing capacity of the delivery orders is of 50%, respectively at least 2 logistic centers and 2 distribution centers work at 50% of the capacity;
- -in the first operational status, when at the downstream level of the logistic system the processing capacity of the delivery orders is of 30%, respectively at least one logistic centers and 2 distribution centers work at 30% of the capacity;
- in the zero operational status, when at the downstream level of the logistic system the processing capacity of the delivery orders is under 30%.

Table 1 The distribution of the probabilities of status of the logistic system from the perspective of the suppliers' operational status

Status *j*		**The probability of a logistic system to be in the operational status *j***	**The probability of a logistic system to be in the operational status *j* or in operational statuses superior to it**
3	The logistic system has the capacity of assuring 100% of the received orders	$r_{s,3}$ = 22,65 %	Pr{ϕ(x)=3} = 22,65 %
2	The logistic system has the capacity of assuring 80% of the received orders	$r_{s,2}$ = 37,64 %	Pr{ϕ(x)□2} = 60,30 %
1	The logistic system has the capacity of assuring 50% of the received orders	$r_{s,1}$ = 30,05 %	Pr{ϕ(x)□1} = 90,35 %
0	The logistic system has the capacity of assuring below 50% of the received orders	$r_{s,0}$ = 9,64 %	Pr{ϕ(x)□0} = 100 %

The situation of the logistic centers and of the distribution centers, according to the operational statuses, is: $k_1=3$; $k_2=4$; $k_3=5$, $k_4=7$ with $k_1 < k_2 < k_3 < k_4$.

We notice that our system is a k/n:G system with several statuses, increasing, respectively k/7:G with 5 statuses, increasing.

We consider the component of the logistic system to be independent, having the following distribution of the status probabilities; $p_0 = 0,05; p_1 = 0,15; p_2 = 0,10; p_3 = 0,15; p_4 = 0,55$.

The probabilities P_j, that the organizations from inside the logistic system to be in the operational status j or in operational statuses superior to it, are:

$$P_1 = \sum_{j=1}^{4} p_j = 0,95;\ P_2 = \sum_{j=2}^{4} p_j = 0,80;\ P_3 = \sum_{j=3}^{4} p_j = 0,70;\ P_4 = p_4 = 0,55$$

The probabilities Q_j, that the organizations from inside the logstic system to be in the operational status j or in operational statuses superior to it, are:

$$Q_4 = 1 - P_4 = 0,45;\ Q_3 = 1 - P_3 = 0,30;\ Q_2 = 1 - P_2 = 0,20;\ Q_1 = 1 - P_1 = 0,05$$

In the forth operational case, we have: j=4, $k_4=7$.

Applying in a proper way the ratio (14), we obtain the probability that the logistic system to be in the forth operational status:

$$R_{s,4} = \sum_{k=7}^{7} C_7^7 \cdot P_4^7 \cdot Q_4^0 = 0,55^7 = 0,0152.$$

Proceeding in an analogous way, we obtain the following status probabilities for the logistic system considered to be in the mentioned operational status or in operational statuses superior to it:

- for the third operational status: j=3, $k_3=5$,

$$R_{s,3} = \sum_{k=5}^{7} C_5^7 \cdot P_3^k \cdot Q_3^{7-k} = \sum_{k=5}^{7} C_5^7 \cdot 0,70^k \cdot 0,30^{7-k} = 0,6470$$

- for the second operational status: j=2, $k_2=4$,

$$R_{s,2} = \sum_{k=4}^{7} C_7^k \cdot P_2^k \cdot Q_2^{7-k} = \sum_{k=4}^{7} C_7^k \cdot 0,80^k \cdot 0,20^{7-k} = 0,9666$$

- for the first operational status: j=1, $k_1=3$,

$$R_{s,1} = \sum_{k=3}^{7} C_7^k \cdot P_1^k \cdot Q_1^{7-k} = \sum_{k=3}^{7} C_7^k \cdot 0,95^k \cdot 0,05^{7-k} = 0,9962$$

- for 0 operational status: $R_{s,0} = 1$

Using the ratio (15), we calculate the probabilities of the logistic system to be only in the mentioned statuses:

- for the forth operational status:

$$r_{s,4} = R_{s,4} = 0,0152$$

- for the third operational status:

$$r_{s,3} = R_{s,3} - R_{s,4} = 0,6470 - 0,0152 = 0,6318$$

- for the second operational status:

$$r_{s,2} = R_{s,2} - R_{s,3} = 0,9666 - 0,6470 = 0,3196$$

- for the first operational status:

$$r_{s,1} = R_{s,1} - R_{s,2} = 0,9962 - 0,9666 = 0,0296$$

- for zero operational status:

$$r_{s,0} = R_{s,0} - R_{s,1} = 1 - 0,9962 = 0,0038$$

The obtained results are centralized in the table no. 2.

Table 2 The distribution of the probabilities of the logistic system from the perspective of the operational status of the logistics centers and the distribution centers

Status *j*		**The probability of a logistic system to be in the operational status *j***	**The probability of a logistic system to be in the operational status *j* or in operational statuses superior to *j***
4	The logistic system's capacity of processing the received orders is maximal (100%)	$r_{s,4}$ = 1,52 %	Pr{ϕ(x)=4} = 1,52 %
3	The logistic system's capacity of processing the received orders is of 80%.	$r_{s,3}$ = 63,18 %	Pr{ϕ(x)□3} = 64,70 %
2	The logistic system's capacity of processing the received orders is of 50%.	$r_{s,2}$ = 31,96 %	Pr{ϕ(x)□2} = 96,66 %
1	The logistic system's capacity of processing the received orders is of 30%	$r_{s,1}$ = 2,96 %	Pr{ϕ(x)□1} = 99,62 %
0	The logistic system's capacity of processing the received orders is below 30%	$r_{s,0}$ = 0,38 %	Pr{ϕ(x)□0} = 100 %

In order to exemplify the calculating of the reliability and the way of determination of the structure-function of a logistic system we'll transform the organizational scheme from the figure no. 1, in the unitary graph, presented in figure no. 2.

In order to simplify the analysis, the logistic system was discomposed in two subsystems: the supplying system A and the delivery system G. In its turn, the delivery subsystem was decomposed in two components, D and E, each of them being formed of two assemblies, respectively by a logistic center and a grouping of distribution centers.

The reliability of the logistic system R_{Slog}, we'll be determined on the basis of the calculus ratios (1), (2), (4) and (5), calculating each model's reliability. We'll obtain, this way:

$$R_A = 1 - Q_A = 1 - \prod_{i=1}^{7} Q_{f_i} = 1 - \prod_{i=1}^{7}\left(1 - R_{f_i}\right) = 1 - \left(1 - R_{f_1}\right)\left(1 - R_{f_2}\right)\cdot\ldots\cdot\left(1 - R_{f_7}\right)$$

$$R_B = 1 - Q_B = 1 - \prod_{i=1}^{2} Q_{d_i} = 1 - \left(1 - R_{d_1}\right)\left(1 - R_{d_2}\right)$$

$$R_D = R_{h_1}\cdot R_B = R_{h_1}\cdot\left(1 - \prod_{i=1}^{2} Q_{d_i}\right) = R_{h_1}\cdot\left[1 - \left(1 - R_{d_1}\right)\left(1 - R_{d_2}\right)\right]$$

$$R_E = R_{h_2}\cdot R_C = R_{h_2}\cdot\left(1 - \prod_{i=3}^{5} Q_{d_i}\right) = R_{h_2}\cdot\left[1 - \left(1 - R_{d_3}\right)\left(1 - R_{d_4}\right)\left(1 - R_{d_5}\right)\right]$$

$$R_G = 1 - Q_G = 1 - \left(1 - R_D\right)\left(1 - R_E\right) =$$

$$= 1 - \left\{1 - R_{h_1}\cdot\left[1 - \left(1 - R_{d_1}\right)\left(1 - R_{d_2}\right)\right]\right\}\cdot\left\{1 - R_{h_2}\cdot\left[1 - \left(1 - R_{d_3}\right)\left(1 - R_{d_4}\right)\left(1 - R_{d_5}\right)\right]\right\}$$

$$R_{S\log} = R_A\cdot R_G = \left[1 - \left(1 - R_{f_1}\right)\left(1 - R_{f_2}\right)\cdot\ldots\cdot\left(1 - R_{f_7}\right)\right]\cdot\left\{1 - \left\{1 - R_{h_1}\cdot\left[1 - \left(1 - R_{d_1}\right)\left(1 - R_{d_2}\right)\right]\right\}\cdot\right.$$

$$\left.\cdot\left\{1 - R_{h_2}\cdot\left[1 - \left(1 - R_{d_3}\right)\left(1 - R_{d_4}\right)\left(1 - R_{d_5}\right)\right]\right\}\right\}$$

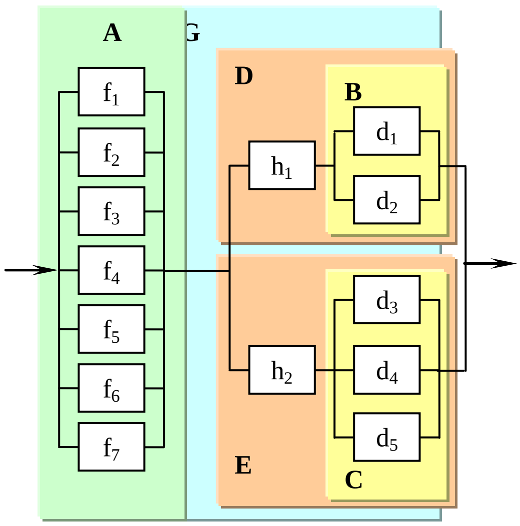

Figure 2 The unitary scheme of the logistic system

If f_i, h_j, d_k represents the statuses of each component organization *i, j, k* respectively, for $1 \le i \le 7$, $1 \le j \le 2$, $1 \le k \le 5$ and

$$f_i, h_j, d_k = \begin{cases} 1 \text{ if the component organization i, j, respectively k is operational} \\ 0 \text{ if the component organization i, j, respectively k isnt operational} \end{cases}$$

(the conditions don't have to be met in the same time by all the component organizations), then the vectors $\mathbf{f} = (f_1, f_2, \ldots, f_7), \mathbf{h} = (h_1, h_2), \mathbf{d} = (d_1, d_2, d_3, d_4, d_5)$ represent the status vectors of the component organizations of the presented logistic system, which influence its operational capacity. Accordingly, the status of the logistic system is a function which is determined by the statuses of the component organizations, given by the ratio:

$$\phi = \phi(\mathbf{f}, \mathbf{h}, \mathbf{d}) = \phi(f_1, f_2, \ldots, f_7, h_1, h_2, d_1, d_2, d_3, d_4, d_5)$$

$i = 1, 2, \ldots, 7$, $j = 1, 2$, $k = 1, 2, \ldots, 5$.

With the help of the ratios (3) and (6), we'll determine the structure-function of the modules represented in the figure no.2 and with their help we can eventually determine the function-structure of the entire logistic system, ϕ_{Slog}.

$$\phi_A(\mathbf{f}) = [1 - (1 - f_1) \cdot (1 - f_2) \cdot \ldots \cdot (1 - f_7)] = \max_{1 \le i \le 7} f_i = \max\{f_1, f_2, \ldots, f_7\}$$

$$\phi_B(\mathbf{d}) = [1 - (1 - d_1)(1 - d_2)] = \max_{1 \le i \le 2} d_i = \max\{d_1, d_2\}$$

$$\phi_C(\mathbf{d}) = [1 - (1 - d_3)(1 - d_4)(1 - d_5)] = \max_{3 \le i \le 5} d_i = \max\{d_3, d_4, d_5\}$$

$$\phi_D(\mathbf{h}, \mathbf{d}) = h_1 \cdot [1 - (1 - d_1)(1 - d_2)] = \min\{h_1, \max\{d_1, d_2\}\}$$

$$\phi_E(\mathbf{h}, \mathbf{d}) = h_2 \cdot [1 - (1 - d_3)(1 - d_4)(1 - d_5)] = \min\{h_2, \max\{d_3, d_4, d_5\}\}$$

$$\phi_G(\mathbf{h}, \mathbf{d}) = 1 - \{1 - h_1 \cdot [1 - (1 - d_1)(1 - d_2)]\} \cdot \{1 - h_2 \cdot [1 - (1 - d_3)(1 - d_4)(1 - d_5)]\} =$$

$$= \max\{\min\{h_1, \max\{d_1, d_2\}\}, \min\{h_2, \max\{d_3, d_4, d_5\}\}\}$$

$$\phi_{S\log}(\mathbf{f}, \mathbf{h}, \mathbf{d}) = [1 - (1 - f_1) \cdot (1 - f_2) \cdot \ldots \cdot (1 - f_7)] \cdot \{1 - \{1 - h_1 \cdot [1 - (1 - d_1)(1 - d_2)]\} \cdot$$

$$\cdot \{1 - h_2 \cdot [1 - (1 - d_3)(1 - d_4)(1 - d_5)]\}\} =$$

$$= \min\{\max\{f_1, f_2, \ldots, f_7\}, \max\{\min\{h_1, \max\{d_1, d_2\}\}, \min\{h_2, \max\{d_3, d_4, d_5\}\}\}\}$$

Analyzing the structure-functions $\phi_A(\mathbf{f}), \phi_B(\mathbf{d}), \phi_C(\mathbf{d})$, it results that the supplying subsystem and the groups of the distribution centers have the operational capacities determined by the supplier, respectively by each distribution center (from each block) with the highest operational status. In the same time, the operational capacity of each block of the delivery subsystem is determined by the minimal operational capacity of the logistic center or the distribution center with the highest operational status. The same reasoning will be applied for the structure-function of the delivery subsystem , $\phi_G(\mathbf{h},\mathbf{d})$, respectively the one of the entire logistic system, $\phi_{S\log}(\mathbf{f},\mathbf{h},\mathbf{d})$.

If we name $\mathbf{f}_i, \mathbf{h}_i$ şi $\mathbf{d}_i$ the event in such a way that the component organization i to be operational and $\bar{\mathbf{f}}_i, \bar{\mathbf{h}}_i$ şi $\bar{\mathbf{d}}_i$, the opposing event, respectively component organization i not being operational, based on the ratio of reliability of the logistic system R_{Slog}, we calculate the logic function of functionality of the logistic system, which shows us the ratio between the operational capacity of the logistic system and the operational statuses of the component organizations:

$$S_{s\log} = \overline{\bar{f}_1 \cdot \bar{f}_2 \cdot \ldots \cdot \bar{f}_7} \cdot \overline{\overline{h_1 \cdot \overline{\bar{d}_1 \cdot \bar{d}_2}} \cdot \overline{h_2 \cdot \overline{\bar{d}_3 \cdot \bar{d}_4 \cdot \bar{d}_5}}}$$

By analyzing this ratio we infer that the logistic system is operational if and only if at least a supplier, a logistic center and a distribution center with which the logistic center collaborates are operational simultaneously, in figure no. 3 this variant being presented. In the same time, the logistic system becomes completely unoperational if and only if all the suppliers are simultaneously unoperational or if both logistic centers, simultaneously, are unoperational or if all the distribution centers, simultaneously, are unoperational, these variants being presented in the figures no. 4, 5 and 6.

We notice in the same time that the operational status of the logistic centers exert a major influence over the operational capacity of the entire logistic system, through the reduced number and through its serial distribution in the distribution centers.

We consider, for example, that the operational statuses of the logistic systems with the highest probability, resulted from the numerical examples presented, are:

- -the second operational status, when at the upstream system of the logistic system 6 suppliers deliver at least 80% of the production obtained in order to assure minimum 80% of the delivery orders towards the clients (the supplier f_7 isn't operational);
- -the third operational status, when the downstream level of the logistic system has the capacity of processing at least 80% of the delivery orders, respectively at least 2 logistic centers and three distribution centers work at least at 80% of the capacity (the distribution centers d_2 and d_5 aren't operational);

In these circumstances, the structure-function of the logistic system will be:

$$\phi_{S\log}^{80\%}(\mathbf{f},\mathbf{h},\mathbf{d}) = \left[1-(1-f_1)\cdot(1-f_2)\cdot\ldots\cdot(1-f_6)\right]\cdot\left\{1-\left(1-h_1\cdot d_1\right)\cdot\left\{1-h_2\cdot\left[1-\left(1-d_3\right)\left(1-d_4\right)\right]\right\}\right\}$$

and the calculation of the logical functionality logical function:

$$S_{s\log}^{80\%} = \overline{\bar{f}_1 \cdot \bar{f}_2 \cdot \ldots \cdot \bar{f}_6} \cdot \overline{\overline{h_1 \cdot d_1} \cdot \overline{h_2 \cdot \overline{\bar{d}_3 \cdot \bar{d}_4}}}$$

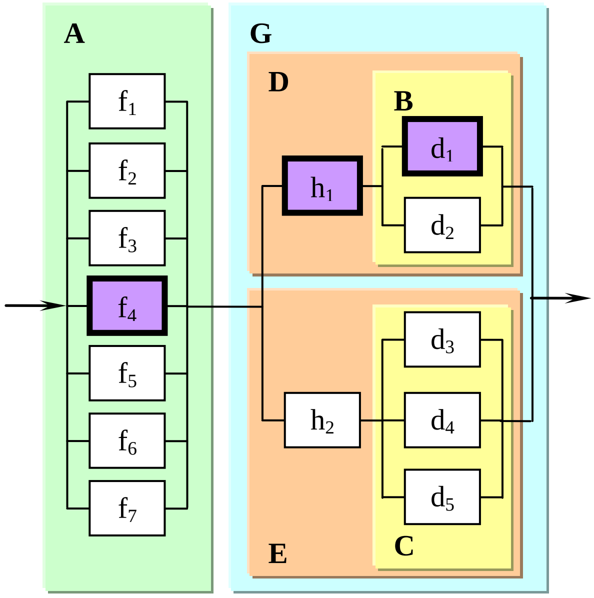

Figure 3 The minimal number of components, categorized on types of components, which is necessary for the a logistic system to be operational

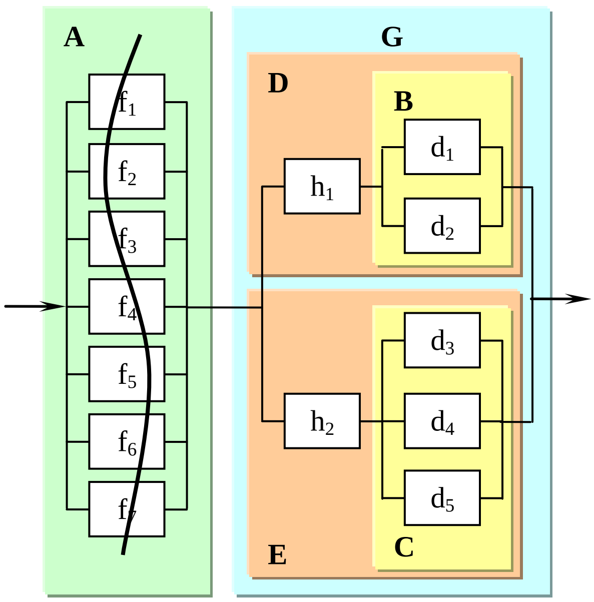

Figure 4 The logistic system isn't operational if all its suppliers aren't operational

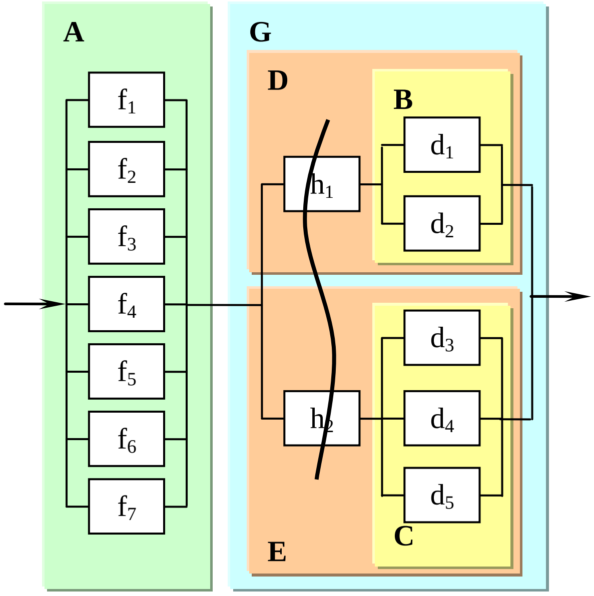

Figure 5 The logistic system isn't operational if its both logistic centers aren't operational

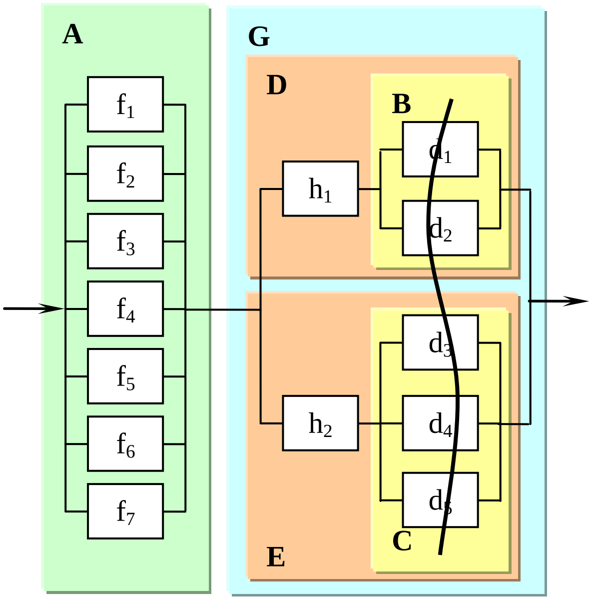

Figure 6 The logistic system isn't operational if its both distribution centers aren't operational

3. Conclusions

A low operational capacity considerably diminishes the level of the sales, increasing the degree of dissatisfaction of the clients, determining an increase in the costs and even endangering the existence of the logistic system.

The structure of the logistic system represents a crucial factor from the point of view of lowering the costs and increasing the quality of services offered to the clients. This substantially influence the operational capacity, respectively the reliability of the processes and the logistic operations that play an essential role in assuring the coherence of the logistic system and implicitly the continuity of the material flows, of the products and data flows. In the same time, next to the efficiency and flexibility, the reliability complements the array of characteristics of an efficient logistics that unfailingly contributes to the increasing of the competitive advantage of the partner organizations.

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