INVESTMENT STRUCTURAL MODELING IN AGRICULTURE

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ABSTRACT

Object of this paper is a problem to be solved that can determine the total volume of investment needed to ensure the completion and operation of new production capacity. Investments, as material support for the economic development, depending on the relation they have with the objective designed, is divided into three categories: a) direct investments or the basic investments are those necessary to achieve the initial capacity. They materialize in expenditure contributing to the achievement itself for the future economic objective, such as costs for the construction of buildings, purchase of machinery, equipment installation, technical evidence etc.; b) collateral investments are those consumption of resources territorial and functional related to the direct investments. Their destination is to create conditions for normal functioning of the basic objective of the project, respectively, to ensure the necessary infrastructure and utilities (access routes, water pipelines, gas pipelines, transmission lines electricity, etc.); c) connected investments are intended to increase production capacity in industries or fields which provides raw materials, energy, fuel, miscellaneous services etc. and preparatory work for the objective in question (consolidation of land, defensive works against floods etc.). These are the investments that we will study in this paper.

KEYWORDS: *investments, modeling, static models, dynamic models*

It is considered a direct (basic) investment, which refers to the company building, producing tractors and agricultural machinery. For this to work, it needs different products (steel industry, chemical industry, timber etc.). These additional claims resulting from direct investments, require increases in production capacity in industries and adjacent areas, so called connected investments.

Direct and collateral investments can be estimated within the investment objective, at the microeconomic level. The connected investments (which liaises between the various activities, branches, which takes place in the national economy), will be calculated separately, using the formula: $I_t = I_d - I_{cn}$, and therefore $I_{cn} = I_t - I_d$, where :

I_t - the total investment volume;

I_d - the volume of direct investments;

I_{cn} - the volume of connected investment.

It is noted that previously we have to determine the total and direct investments and then, according to them, we can determine the connected investments. As we know, specific investment indicator expresses the volume of spending investment that is done to produce one unit of physical production capacity, as follows: $s = \frac{I}{q}$.

Through this indicator we can establish the connection between production capacities to be constructed and investment requirements. In this case, knowing the specific investment and production capacities to be built, we can establish the connected investment related to each industries, using the formula of calculation: $I = s \times q.$ But for this, we must first set out the basic production and production of related industries and only then we can calculate the connected investments.

We must say that for a proper determination of investments in the national economy (macroeconomic) we must known precisely which are the economic relations between the industries. These relationships can be expressed using ”between branches correlations balance”, which expresses the links between inputs and outputs of the national economy, as a multi system. Among the economic applications of this method, we can noted the following: planning the economic development; use of natural resources; technological exchanges and study of the effects on productivity; analysis of the change salaries influence; analysis of the profits and taxes over the prices; the study of international economic relations; forecasting the economic demand; forecasting the production, force employment and investment in the different sectors etc.

In the following we will present the determination of the connected investments related to the macroeconomic level, using input-output methodology. This method was developed by American economist of Russian origin Wassily Leontief and was published in the paper “Input-Output Economics”, Oxford University Press, New York, 1966.

1. Static model of input-output table

A condition for balanced development of industries throughout the national economy is the identification of the main links and interdependencies between them. These links are called links between industry and the mathematical model that describes the links inter-branches of the national economy model is called ”the input-output table”. The model allows characterization and development of the economy as a whole and individual economic sectors, relations between the branches and between different sides of the breeding social setting proportions of industry and development on this basis in a short-term variations. “Input-output table” model is a simple model that explains how the system actually formed production output of the final demand: consumption, investment and export. This model expresses pretty good economic processes taking place in reality and so is a convenient and useful for practical purposes. At the fundament of the model there are some assumptions regarding the properties of the economic system.

1. The economic system (the national economy) is composed by specific objects - economic branches. Each of the branches (sectors) produces a type of production, and the volume can be characterized by a number. Such a number can also be global volume of domestic production in terms of currency units (measure of money in production volume)

or expressed physically (natural calculating the volume of production units, tons, meters, liters etc.).

2. In order to produce the production volume, every branch uses economic strictly determined quantities of production of other branches.

3. The increase of the volume of production for several times requires the increase in consumption by the industry that all other types of products of the same number of times (direct ownership of proportionality).

4. Production made by each branch is partly consumed by other branches (n the sphere of material production) and some go outside as the final product of that system. In the static model of "input-output table" model, W. Leontief assumes that the purpose of the economic system consists in the production of fixed quantities for final products.

"Input-output table" model shall be made in table form, which describes the flow of goods and services between all branches of the national economy over a given period of time, usually a year. This picture has its value, which expresses the conversion unit (value) of the quantities of calculation (input-output) of the table, and may be represented as a system of national accounts.

Production of each branch to the national economy appears as the result of a series of inputs (supplies received from other branches of activity). At the same time, production of each branch is distributed in a number of output sites (deliveries to other economic sectors in order to productive consumption - intermediate consumption - and supplies for personal and social population, and capital funds for building the necessary future production – final consumption). In the first moment, we face with a technique of organizing information-statistical material available for a certain period of time (usually a year) in a table, where rows expresses exits from the system, and columns reflected in the multi entries.

At the national economy level, a growing importance belongs to the proper evaluation of the volume of investment needed for economic development, namely the volume of investment in branches of production and materials sectors, now and in perspective. The volume of investments depends on the volume of production to be achieved and on economic efficiency of the future fixed capital which will be put into operation. In this case, an important place it occupies the establishment of production for each branch and total production throughout the national economy. This can be done, precisely, with the "input-output table" model, which is a method of analysis, modeling of economic phenomena and determination of inter-relationships formed in the national economy.

For the presentation of mathematical "input-output table" model we'll go from the broad picture 'input-output', which structure into four dials, each with specific content and meanings.

First dial - shows on the lines the branches of production "*i*" and on the columns branches consuming "*j*". Sizes written on the lines represent "outputs" and expressed as a distributed part of the total production of equipment for the intermediate consumption of economic branches. Sizes written on the columns appear on the "inputs" and reflect the expenditure materials pattern, to achieve a certain volume of production in that branch. Thus, it shows the links of the production-consumption, interdependence between branches on raw materials, materials, fuel, energy, etc. at the national economy level.

Second dial - the sizes listed on lines reflect how the branches of the national economy contributes to the formation of the final used product (final demand), which implies: unproductive consumption (of the population and administration), productive consumption (accumulation of gross fixed capital or investmen), other forms of consumption (savings of goods and reserves) and applications for export. Sizes listed on the columns expressing the structure of entries on each component of final demand. It follows that this dial indicate the resources available to the national economy in a given period of time and manner of distributing them on the elements of final demand.

Third dial **-** presents on the lines the direction of primary sharing of national income in the branches that occurred at the creation: raw material costs and materials, wages and other forms of remuneration of labor, depreciation and outstanding cost, profit and other forms of additional product. On the columns we can find the value of which these items contribute within the competence of the respective branch (value added, which is calculated as the difference between the production value and the value of intermediate consumption).

Fourth dial **-** expresses the process of reallocation, characterized by the use of national income to consumption and accumulation. In other words, the dial reflects the gross domestic product as a sum of the values added. Unfortunately, this problem has not yet been solved in any country and remains a desideratum of scientific research in the field of this model.

A last dial would indicate the value of total resources of the nation, including the value of imports.

The general scheme of the model is presented in Table 1.

„Input-output table" model, in static vision

Table 1

Production value from the production branches	**Interbranches flows (consumption branches)**	**Final product**
X_1 X_2 … X_i … X_n	$x_{11}\ x_{12}$ .... x_{1j} .... x_{1n} $x_{21}\ x_{22}$ .... x_{2j} .... x_{2n} …………. **First Dial** …..…. $x_{i1}\ x_{i2}$ .... x_{ij} .... x_{in} ………………………………. $x_{n1}\ x_{n2}$ .... x_{nj} .... x_{nn}	Y_1 Y_2 **Second Dial** Y_i … Y_n
Depreciation Wages Overproduction Global product	$a_1\ \ a_2$ .... a_j .... a_n $S_1\ \ S_2$ .... S_j .... S_n **Third Dial** $P_1\ \ P_2$ .... P_j .... P_n $X_1\ X_2$ .... X_j .... X_n	**Fourth Dial**

In the balance of links between the economic branches we can find all the branches of the national economy and each of them appearing twice: once as a producer of production materials and services, and a second time as their consumers. So for "*i*" branch as a producer, it corresponds to the line „*i*" $(i=\overline{1,n})$, and for "*j*" branch as a consumer of production, it corresponds to the column „ j" $(j=\overline{1,n})$, from the table no. 1. At the intersection of the "i" line and "j" column there is the x_{ij} variable, which must be understood as the cost of material production manufactured by the "i" branch and consumed material costs in "j" branch. In each of the "n" columns of the "input-output" table we can see the structure of materials costs, depreciation and net production for each branch. Net production (national income), without income from external trade, is calculated by adding together the salaries and overproduction.

As a consequence of the above considerations, we can develop the mathematical model of "input-output" table, through a system of linear equations to express links according the physical and monetary structure.

We combine elements of each row to obtain the balance equation of sharing products, as follows: $X_i = \sum_{j=1}^{n} x_{ij} + Y_i$, where: $i=\overline{1,\mathrm{n}}$.

Also, we combined elements of each column to obtain the balance equation of costs of production, thus: $X_j = \sum_{i=1}^{n} x_{ij} + a_j + S_j + P_j$ or $X_j = \sum_{i=1}^{n} x_{ij} + VA_j$, where $j=\overline{1,\mathrm{n}}$.

The symbols in the above equations have the following economy meanings :

X_i – the value of production in the branch i;

x_{ij} – the volume of production of the branch i that is consumed in branch j;

Y_i – the value of production in the branch i, which is part of the national income (final product);

X_j – the production costs of branch j;

a_j – depreciation for the replacement of fixed assets in industry j;

S_j – wages in branch j;

P_j – overproduction achieved in the branch j;

VA_j – the added value of branch j.

Therefore, the production of each branch is obtained by adding together the elements of the row or of the column. In this case, the production of the branch i is obtained as follows:

$$X_i = \sum_{j=1}^{n} x_{ij} + Y_i = \sum_{k=1}^{n} x_{ki} + a_i + S_i + P_i \quad \text{where: } i=\overline{1,\mathrm{n}}$$

It is noted that the two amounts are not reduced, because the amount according to i and belonging to x_{ij} is determined on the line, and the amount according to k and belonging to x_{ki} is calculated on the column. They have one common element x_{ii}, and the exclusion of it determine the *balance equation of the flows between the branches*, as follows:

$$X_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} x_{ij} + Y_i = \sum_{\substack{k=1 \\ k\neq i}}^{n} x_{ki} + a_i + S_i + P_i$$

From the above formulas, in order to determine the investments, we are interested in the balance equation of the products allocation, which can be written like this:

$$\begin{cases} X_1 = x_{11} + x_{12} + ... + x_{1j} + ... + x_{1n} + Y_1 \\ X_2 = x_{21} + x_{22} + ... + x_{2j} + ... + x_{2n} + Y_2 \\ .... \\ X_i = x_{i1} + x_{i2} + ... + x_{ij} + ... + x_{in} + Y_i \\ .... \\ X_n = x_{n1} + x_{n2} + ... + x_{nj} + ... + x_{nn} + Y_n \end{cases}$$

Through these formulas, it is reflected equality that should exist between the production of each branch of production (X_i) and the flows of production $(\sum_{j=1}^{n} x_{ij},\ i = \overline{1, n})$, from branch *i* to producing and consuming industries *j*, plus the final product (Y_i) of the production branch. We will need to determine the technical coefficients or proportionality (a_{ij}) of each branch: $a_{ij} = \frac{x_{ij}}{X_j} \Rightarrow x_{ij} = a_{ij} \cdot X_j$ according with the condition $0 \leq a_{ij} \leq 1$.

In this case, the a_{ij} coefficients represent the share of material consumption of branch *j*, from industry *i*, in total production of the branch *j*. So, these coefficients show how many units of the total production of manufacturing industry *i* are spent to produce a unit of measurement of the production in the consuming branch *j*. They are also called *consumption coefficients*.

As a result, balance the equation of the products allocation, taking into account the constant values (a_{ij}), becomes $X_i = \sum_{j=1}^{n} a_{ij} \cdot X_j + Y_i$, where $i = \overline{1, \text{n}}$.

In the form of linear equations, the above formula can be written as follows:

$$\begin{cases} X_1 = a_{11} \cdot X_1 + a_{12} \cdot X_2 + ... + a_{1j} \cdot X_j + ... + a_{1n} \cdot X_n + Y_1 \\ X_2 = a_{21} \cdot X_1 + a_{22} \cdot X_2 + ... + a_{2j} \cdot X_j + ... + a_{2n} \cdot X_n + Y_2 \\ .... \\ X_i = a_{i1} \cdot X_1 + a_{i2} \cdot X_2 + ... + a_{ij} \cdot X_j + ... + a_{in} \cdot X_n + Y_i \\ .... \\ X_n = a_{n1} \cdot X_1 + a_{n2} \cdot X_2 + ... + a_{nj} \cdot X_j + ... + a_{nn} \cdot X_n + Y_n \end{cases}$$

This equations system means exactly the “input-output table” model of Leontief. From the historic point of view, it is the first and most simple model “input-output” model, available for practical calculations.

If we note A - direct consumption matrix, with X - column vector of the industries and Y - column vector of the final production, the above system becomes:

$X = A \cdot X + Y$, where: $X = \begin{Vmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_n \end{Vmatrix}$, $Y = \begin{Vmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_i \\ \vdots \\ Y_n \end{Vmatrix}$, $A = \begin{Vmatrix} a_{11}\ a_{12} \ldots a_{1j} \ldots a_{1n} \\ a_{21}\ a_{22} \ldots a_{2j} \ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{i1}\ a_{i2} \ldots a_{ij} \ldots a_{in} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}\ \mathrm{a}_{n2} \ldots a_{nj} \ldots a_{nn} \end{Vmatrix}$

In the calculations, in order to determine the total investment, we are interested in total consumption matrix, which in literature is known as the inverse matrix of Leontief, and has the form: $(E-A)^{-1}$. The calculation of this matrix is established by successive methods, namely $(E-A)^{-1} = E + A + A^2 + A^3 + \ldots$, where: E – unit matrix and is as follow:

$$E = \begin{Vmatrix} 1 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & \ldots & 1 \end{Vmatrix}$$

The inverse matrix of Leontief may be defined as the total quantity of products of the manufacturing sector, which is consumed to produce one unit of the final product of the consuming branch *j*. Coefficients for the inverse matrix can be written like this:

$$(E-A)^{-1} = \begin{Vmatrix} A_{11}\ A_{12} \ldots A_{1j} \ldots A_{1n} \\ A_{21}\ A_{22} \ldots A_{2j} \ldots A_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A_{i1}\ A_{i2} \ldots A_{ij} \ldots A_{in} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A_{n1}\ A_{n2} \ldots A_{nj} \ldots A_{nn} \end{Vmatrix}$$

To establish the matrix of total consumption, we'll go from the matrix formula of the basic relation $(X = A \cdot X + Y)$, which can also be written as follow $E \cdot X = A \cdot X + Y$ or $E \cdot X - A \cdot X = Y$, and thus $(E-A) \cdot X = Y$ or $X = (E-A)^{-1} \cdot Y$. Therefore, using the inverse matrix, we can obtain the total quantity of production, based on the formula: $X = (E-A)^{-1} \cdot Y$, and therefore we will obtain column vector of the total production of each producing branch *i*:

$$\begin{Vmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_n \end{Vmatrix} = \begin{Vmatrix} A_{11}\ A_{12} \ldots A_{1j} \ldots A_{1n} \\ A_{21}\ A_{22} \ldots A_{2j} \ldots A_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A_{i1}\ A_{i2} \ldots A_{ij} \ldots A_{in} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A_{n1}\ A_{n2} \ldots A_{nj} \ldots A_{nn} \end{Vmatrix} \cdot \begin{Vmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_i \\ \vdots \\ Y_n \end{Vmatrix}$$

The calculation shows us that the total production of an industry (X) depends on the total consumption, illustrated by the inverse matrix of Leontief $(E-A)^{-1}$ and final consumption (Y). We remind that the multiplication is not commutative in the matrix calculations, and therefore we should keep the formula: $X=(E-A)^{-1}\cdot Y$.

In addition, the analyzed model can be filled with restrictions according to the use of production factors, which refers to the natural resources, labor resources and fixed capital. If we assume that the costs of production factors are proportional with the production volumes and the volumes of production factors are limited, we will obtain the additional linear restrictions, as follows:

$$\sum_{j=1}^{m} c_{\kappa j}\cdot X_j \le v_{\kappa}$$

where:

$c_{\kappa j}$ - the direct cost of production factor κ for the manufacturing one unit of production of *j* type;

X_j - cost of production of *j* branch;

v_{κ} - volume of production factor.

Once studied these basic elements, we could continue to establish total investments (direct and connected) corresponding to a specific production increase. In order to determine the investments needed to achieve total production, we have to start from the specific investment, calculated using the formula $s=\frac{I}{q}$.

Taking into account that the calculations are made at the macroeconomic level, where indicators are expressed in monetary units, we'll determine the specific investment according the value of production, namely:

$$b=\frac{K}{V}=\frac{I}{V} \text{ or } s=\frac{I}{Q} \text{ and thus } I=s\cdot Q$$

where:

b - Harrod's capital coefficient;
K - the amount of fixed capital;
-V income / national product made;
Q - the value of production.

Through this last formula, we noticed that the volume of investment depends on the specific investment and production to be achieved. In the “input-output table” model the production is marked with *X*, which means that the formula $I=s\cdot Q$ can also be written $I=s\cdot X$ or $I=s\cdot(E-A)^{-1}\cdot Y$.

Therefore, the volume of investment depends on the specific investment, total consumption (expressed by the inverse matrix of Leontief) and the final product to be achieved. As the total consumption matrix is marked with $(E-A)^{-1}$, if we mark specific direct investment with s_d, we will get the formula: $I=s_d\cdot(E-A)^{-1}\cdot Y$.

We can define specific direct investment as investment expense, expressed in monetary units, which is done by direct investment in each economic branch, in order to obtain a monetary unit of the final product in the same branch. So for example, if specific direct investment in branch *A* is 0.87, this means that in the branch *A* we have to spent 0.87

monetary units in order to obtain one monetary unit of final product. In this case, direct investments depend on the specific direct investment and the final product, as follows:

$$I_d = s_d \cdot Y$$

where:

I_d - direct investment;

s_d - specific direct investment;

Y - the final product.

Depending on the specific direct investment and total consumption, we can calculate the total specific investments, namely: $s_t = s_d \cdot (E - A)^{-1}$, where: s_t - specify the total investment.

Thus calculated, specific total investment reflects how much is spent in all branches of the national economy to achieve a monetary unit of the final product in a particular branch. In this case, the total investment depend on the specific total investment and the final product, as follows: $I_t = s_t \cdot Y$ or $I_t = s_d \cdot (E - A)^{-1} \cdot Y$ where: I_t - total investment.

Based on the above formulas, it shows that for establish the necessary investment (direct, connected and total), we should undergo several stages, namely:

1. determine the coefficients of the direct consumption or direct expenditure (a_{ij}) on the basis of statistical data from the "input-output table" model;

2. calculate the total expenditure, illustrated by the inverse matrix $(E - A)^{-1}$ of Leontief, based on the consumption coefficients, A matrix;

3. based of specific direct investment (s_d) and total costs matrix $[(E - A)^{-1}]$, we may establish total specific investment $[s_t = s_d \cdot (E - A)^{-1}]$;

4. the final product (Y) to be developed in each branch of the national economy will be determined;

5. direct investment is calculated, for each related economic branches, as follows: $I_t = s_t \cdot Y$;

6. total investment will be determined $I_t = s_t \cdot Y$, and also connected investment $I_{cn} = I_t - I_d$;

7. the specific connected investment (s_{cn}) will be determined, as a difference between specific total investment (s_t) and specific direct investment (s_d), as follow : $s_{cn} = s_t - s_d$.

Before to conclude the study of the static model of Leontief, we must say that the direct investment can also be determined through the estimated expenditure list for every investment objective.

2. Dynamic models

Until now the economic and mathematical model has been studied, which express the links between branches of the national economy at a given period of time. In this model, we were not taken into account elements such as the evolution in time of the phenomenon, changes in the structure of industries, the effects of technical progress over the production and efficiency investments. Thus, we can't say that the model has a dynamic vision.

However, it is known that the economy is a dynamic system, meaning that its component elements are developed over time. One of the effective methods to study the dynamic of economics, both in the theoretical and the practical aspect, is the dynamic "input-output table" model, which is further development of the static model. This model is designed to express the development economic process, to determine immediate link between the precedent and successive stages of development. Mathematical dependence between the volume of investment capital and production increase serves as the basis for establish various versions of models of dynamic "input-output table" model. A big contribution in the development and study of economic growth, based on this type of models, belongs to the scientists Wasily Leontieff and Oskar Lange. As a result, we intend to examine briefly these two models.

The dynamic model of W. Leontieff

To develop the dynamic model, Leontieff has dismantled the final product into two components, namely:

- part of the production for investment, which will lead to the creation of new machinery, plant work etc. or to modernize and develop existing ones;
- the rest of production, which is designed for population consumption and export.
- In these conditions, it results the following formula:

$$Y_i = K_i + Y_i' \quad , \qquad i = 1, 2, 3, \ldots, n$$

in which:

Y_i - final production in branch *i*;

K_i - part of the production if the branch *i*, for the investment;

Y_i' - final demand of the branch *i*, for export and consumption.

For the static model the equation was written as follows:

$$X_i = \sum_{j=1}^{n} x_{ij} + Y_i \ , \quad i = 1, 2, 3, \ldots, n$$

in which:

X_i - the total production of manufacturing industry *i*;

x_{ij} - part of the production of the branch *i*, designated for productive consumption in the branch *j*;

Y_i - the value of production for the manufacturing sector *i*, designated for the final product.

Returning to the Leontieff dynamic model, the above relationship becomes:

$$X_i = \sum_{j=1}^{n} x_{ij} + \sum_{j=1}^{n} K_{ij} + Y_i' \ , \quad i = 1, 2, 3, \ldots, n$$

where: K_{ij} - part of the production of the productive sector *i*, designated for investment in the consumption sector *j*;

Y_i' - the final product without investment (also called final demand).

Thus, the above formula proves that the total production of manufacturing industry *i* (X_i) is divided for productive consumption of raw materials, fuel, energy in consumption industries *j* (x_{ij}), the investment to be completed in consumption branch *j* (K_{ij}) and export and people consumption, namely final demand (Y_i').

The simplified scheme of the dynamic "input-output table" model of Leontieff is as follow:

Dynamic „input-output table" model of Leontieff

Table. 2

Productive branches	Production flows toward the consumption branch	Material good flows designated for investments in the consumption branches	Final demand
X_1 X_2 … X_i … X_n	$x_{11}\ x_{12}$ .... x_{1j} .... x_{1n} $x_{21}\ x_{22}$ .... x_{2j} .... x_{2n} .... **First Dial** …….. $x_{i1}\ x_{i2}$ .... x_{ij} .... x_{in} .... $x_{n1}\ x_{n2}$ .... x_{nj} .... x_{nn}	$K_{11}\ K_{12}$ .... K_{1j} .... K_{1n} $K_{21}\ K_{22}$ .... K_{2j} .... K_{2n} .... **Second Dial** ……… $K_{i1}\ K_{i2}$ .... K_{ij} .... K_{in} .... $K_{n1}\ K_{n2}$ .... K_{nj} .... K_{nn}	Y_1' Y_2' … Y_i' … Y_n'
Total	$X_1\ X_2$ .... X_j .... X_n	$X_1\ X_2$ .... X_j .... X_n	

In this table, it was represented :

- in the first dial (the consumption of raw materials, fuel, energy, etc. from the consumption branches (x_{ij}));
- in the second dial (the consumption of material goods for investment in consumption branches (K_{ij}));
- in the first column (productive industries (X_i));
- in the last column (final demand (Y_i'));
- in the last row (the total production of consumption branch (X_j)).

Let's go back again to the static model of Leontieff: $x_{ij} = a_{ij} \cdot X_j$, based on the $a_{ij} = \frac{x_{ij}}{X_j}$.

Similarly, investment technical coefficients can be calculated :

$$b_{ij} = \frac{K_{ij}}{X_j}\text{, and thus } K_{ij} = b_{ij} \cdot X_j$$

Thus, investments coefficients (b_{ij}) shows part of production in manufacturing industry *i*, which must be invested in the consumption branch *j*, in order to increase production volume of this branch with one unit.

If we will take a look at Harrod model, we will noticed that $b = \frac{K}{V}$ (*V* is nothig else than *X*). Therefore, the dynamic model of Leontief can also be written in the following way:

$$X_i = \sum_{j=1}^{n} a_{ij} \cdot X_j + \sum_{j=1}^{n} b_{ij} \cdot X_j + Y_i' \quad , \qquad i = 1, 2, 3, \ldots, n$$

But, the above formula does not quantify the time factor. Therefore we consider the formula: $K_{ij} = b_{ij} \cdot X_j$ as to be continued over time and therefore can be differentiated according the time, namely: $\frac{dK_{ij}}{dt} = b_{ij} \cdot \frac{dX_j}{dt}$. In this circumstances, the relationship of inter-branch balance from above will become: $X_i = \sum_{j=1}^{n} x_{ij} + \sum_{j=1}^{n} \frac{dK_{ij}}{dt} + Y_i'$.

Thus, we obtain a relationship which shows that the output (X_i) of the manufacturing branch *i* switches - in a given period - to the consumption branch *j*, as a part of the production of this industry (X_j), through two pathways, namely:

- on the form of direct production flows (x_{ij} - raw materials, fuel, energy, etc.);
- on the form of a part of the operational investments $\left(\frac{dK_{ij}}{dt}\right)$.

As a result, the final shape of the Leontieff dynamic model, expressed as a system of *n* linear differential equations of the first degree, is as follow:

$$X_i = \sum_{j=1}^{n} a_{ij} \cdot X_j + \sum_{j=1}^{n} b_{ij} \cdot \frac{dX_j}{dt} + Y_i' \quad , \qquad i = 1, 2, 3, \ldots, n$$

where: X_i - column vector of total production in manufacturing industries *i*;

X_j - column vector of total production in the consumption sectors *j*;

Y_i' - column vector of final demand made in branch *i*;

a_{ij} - consumption coefficients matrix;

b_{ij} - investments coefficients matrix.

In the matrix form, Leontieff dynamic model can be written as follow:

$$X_i = \begin{Vmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_n \end{Vmatrix}, \quad X_j = \begin{Vmatrix} X_1 \\ X_2 \\ \vdots \\ X_j \\ \vdots \\ X_n \end{Vmatrix}, \quad Y_i' = \begin{Vmatrix} Y_1' \\ Y_2' \\ \vdots \\ Y_i' \\ \vdots \\ Y_n' \end{Vmatrix}, \quad A = \begin{Vmatrix} a_{11}\ a_{12} \ldots a_{1j} \ldots a_{1n} \\ a_{21}\ a_{22} \ldots a_{2j} \ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{i1}\ a_{i2} \ldots a_{ij} \ldots a_{in} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}\ a_{n2} \ldots a_{nj} \ldots a_{nn} \end{Vmatrix}, \quad B = \begin{Vmatrix} b_{11}\ b_{12} \ldots b_{1j} \ldots b_{1n} \\ b_{21}\ b_{22} \ldots b_{2j} \ldots b_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ b_{i1}\ b_{i2} \ldots b_{ij} \ldots b_{in} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ b_{n1}\ b_{n2} \ldots b_{nj} \ldots b_{nn} \end{Vmatrix}$$

Knowing coefficients of consumption matrix *A*, the investment coefficients matrix *B*, the column vector of final demand (Y_i'), we may determine the total production to be carried out in all the connected sectors, expressed by vector column of the productive industries (X_i) consumption industries (X_j).

The system of linear differential equations of Leontieff is known with two variants, namely: the dynamic closed open and dynamic system.

- *in the closed dynamic system*, final demand is involved in the total production ($Y_i' = 0,$), and the final form of the model is as follow

$$X_i - \sum_{j=1}^{n} a_{ij} \cdot X_j - \sum_{j=1}^{n} b_{ij} \cdot \frac{dX_j}{dt} = 0 \quad , \qquad i = 1, 2, 3, \ldots, n$$

- *in the open dynamic system*, final demand is explicitly illustrated ($Y_i' > 0,$) and the final form of the model is as follow :

$$X_i - \sum_{j=1}^{n} a_{ij} \cdot X_j - \sum_{j=1}^{n} b_{ij} \cdot \frac{dX_j}{dt} = Y_i' \quad , \qquad i = 1, 2, 3, \ldots, n$$

Conclusion

In conclusion, the “input-output table” model, both in static and dynamic form, reflects the links between “inputs” and “outputs”. Obviously, a more rigorous identification of inputs, a proper evaluation of them will lead to more accurate expression of the outputs of the system (indicators performance). However, difficulties arise in determining these flows coefficients relating to production and investment, because on the one hand, they fluctuate yearly, and on the other hand, their size affects the estimation of indicators (production, fixed capital etc.) for the period future (which can occur in significant changes in terms of structure and the volume of output).

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